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ABSTRACT

The dimensionality of one form of the Law School Admission Test (LSAT) was assessed with respect to three ethnic groups of test takers. Whether differences in the ability composite have any noticeable impact on item response theory (IRT) true score equating results for these subgroups (African Americans, Hispanic Americans, and Whites) was also studied. Results obtained with respect to the dimensionality of the LSAT showed that a two-dimensional model, specifying analytical reasoning and logical reasoning plus reading comprehension as two abilities, adequately accounted for the item responses of both African-American and Caucasian test takers, but a more complex model was required for the Hispanic subgroup. Results obtained in this study suggest that African-American and Hispanic-American conversion lines appear to be equivalent to the equating function of the majority Caucasian group as well as to the one derived from the total test-taker population. In other words, the current practice of applying a conversion function obtained from the total population to all test takers, without regard to ethnicity, does not penalize minority group test takers. Five tables and 15 figures present results of the analyses. (Contains 71 references.) (SLD)

Assessing the Effect of Multidimensionality on IRT True-Score Equating for Subgroups of Examinees

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Abstract

The purpose of this study was to assess the dimensionality of one form of the LSAT with respect to three ethnic groups of test takers and to investigate whether differences in the ability composite have any noticeable impact on IRT true-score equating results for these subgroups. More precisely, the conversions estimated for African-American and Hispanic test takers were compared to the equating functions derived from the majority (Caucasian) group as well as the total test taker population to see if there existed any noteworthy differences.

Results obtained with respect to the dimensionality of the LSAT with the three ethnic groups showed that a two-dimensional model, specifying Analytical Reasoning and Logical Reasoning + Reading Comprehension as two abilities, adequately accounted for the item responses of both Caucasian and African-American test takers whereas a more complex model was required for the Hispanic subgroup.

Equating results indicated that the differences between the conversion lines obtained for the three ethnic groups and the total test taker population were negligible. The largest residuals obtained when comparing the minority group conversion lines to either the Caucasian or total population equating functions were well within one conditional standard error of measurement for score differences which again would indicate that the variations are of little practical significance.

Also, the effect of matching Caucasian test takers on the basis of the African-American raw-score frequency distribution did tend to increase the disparities between the equating functions at the extremes, hence contributing to a slightly larger mean absolute residual value. However, the discrepancies between the two conversion lines in the middle of the scale were smaller. These findings support those of Cook, Eignor, and Schmitt (1990) as well as Kolen (1990) who stated that matching generally did not contribute to a more accurate equating.

Regardless, the results obtained in this study suggest that African-American and Hispanic conversion lines appear to be equivalent to the equating function of the majority Caucasian group as well as to the one derived from the total test taker population. In other words, the current practice of applying a conversion function obtained from the total population to all test takers, irrespective of ethnicity, does not penalize minority test takers.

Introduction

As the Law School Admission Test (LSAT) is usually disclosed after each national administration, the Law School Admission Council (LSAC) prefers not to re-administer the same test form repeatedly to different groups of examinees. Therefore, several test forms of the LSAT are constructed to be as similar as possible with regard to the content of the items, the skills being targeted as well as the statistical characteristics of both the items and the test. This is accomplished by developing test items and forms that conform to a well-defined set of content as well as statistical specifications. However, in practice, LSAT test forms do vary slightly with respect to their statistical attributes. This fact makes it impossible to compare scores across different forms without adjusting for these differences. This adjustment is done through a process called *score equating*. Lord (1977) states that two test forms are equated when it is a matter of indifference to each examinee, or to anyone using the results, which test form he or she takes. More precisely, Lord (1980) states that scores on two tests can be equated if they show evidence of meeting the following four conditions:

- *Unidimensionality*, that is, the same underlying construct must be present in both test forms;
- *Equity*, that is, for each group of examinees of identical ability, the conditional frequency distribution of scores on one form (e.g., *Y*), after transformation, must be the same as the conditional frequency distribution of scores on the other test (e.g., *X*);
- *Population invariance*, that is, the equating function must be independent of the group from which it was derived;
- *Symmetry*, that is, the equating is transposable. The function that transforms scores from Form *X* to Form *Y* is the same as the one that maps scores from Form *Y* to Form *X*.

The conditions of unidimensionality and population invariance set out by Lord (1980) are especially relevant for IRT-based equating procedures. One of the major theoretical advantages of IRT, within an equating application, is that the function derived to transform scores from one test form to another should be independent of the population on which it was based (Cook & Petersen, 1987). However, this property does not hold if the assumptions of the models, one of which is unidimensionality of the latent ability space, are violated. Common IRT models (Hambleton & Swaminathan, 1985; Hulin Drasgow, &

Parsons, 1983) assume that item response probabilities are a function of a single latent ability. One such model is the three-parameter logistic function, given by,

$$P_i(u_i=1|\theta_j) = c_i + (1-c_i)\ \frac{e^{Da_i(\theta_j-b_i)}}{1+e^{Da_i(\theta_j-b_i)}}. \qquad (1)$$

This function specifies the probability that a randomly selected examinee of ability level θ_j will correctly answer a given dichotomous item i. The item difficulty parameter or b_i, corresponds to the ability value at the point of inflexion of the item characteristic curve (ICC) whereas the discrimination parameter or a_i, is the value of the slope of the ICC at its point of inflexion. The lower asymptote parameter or c_i, corresponds to the minimum $P(\theta_j)$ value, sometimes called the *pseudo-guessing* parameter. The value D is a constant used to approximate the normal ogive model (≈ 1.7). Although most IRT models assume that item response probabilities can be estimated in a unidimensional space, this condition is rarely met in practice (Traub, 1983). A mathematics test, for example, might entail not only mathematical ability but also the capability to read and understand the problems being presented. Hence, the advantages of using IRT models to equate scores on two test forms, namely the population invariance property, might not generalize to conditions where the assumption of unidimensionality has been compromised. This led Petersen, Kolen, & Hoover (1989) to state that the unidimensionality and population invariance conditions set out by Lord (1980) are intertwined. Equating functions derived from tests that measure different abilities will probably vary for different groups of examinees (Petersen, Kolen & Hoover, 1989). Indeed, Lord and Novick (1968) as well as Bejar (1983) have stressed that dimensionality must be viewed as an interaction of a given sample of examinees with a specific set of items rather than solely as a characteristic of the content of the test.

The effect of multidimensionality on equating

Several researchers have attempted to assess the effect of multidimensionality on IRT true-score equating functions (Bogan & Yen, 1983; Camilli, Wang, & Fesq, 1992; Cook & Douglass, 1982; Cook, Dorans, Eignor, & Petersen, 1985; Dorans & Kingston, 1985; Kolen & Whitney, 1982; Modu, 1982; Snieckus & Camilli, 1993; Stocking & Eignor, 1986; Wang, 1985; Yen, 1984). A comprehensive review of these papers can be found in Harris (1993). The majority of these studies concluded that although multidimensionality of the latent ability space did affect the quality of IRT true-score equating, the

impact often appeared to be minimal and of little practical significance. These findings confirm an earlier statement made by Digvi (1981) to the effect that IRT applications that deal with entire tests such as equating, are more likely to be robust to departures from model assumptions, e.g., unidimensionality of the latent ability space.

Kolen & Whitney (1982) found that differences in the dimensionality of various tests of General Educational Development (GED) did not affect IRT true-score equating results in a noticeable fashion. Bogan & Yen (1983) and Yen (1984), in vertical equating studies, similarly concluded that unsystematic errors of equating are to be expected when equating two multidimensional tests. However, substantial systematic errors might be expected solely when attempting to equate two tests that measure very discrepant ability composites. Wang (1985) and Goldstein and Wood (1989) also stated that the impact of multidimensionality on the quality of IRT equatings is likely to be negligible as long as the same linear composite of abilities underlies the item responses on both tests.

Dorans & Kingston (1985), compared conversion tables based on calibrations of homogeneous Graduate Record Examinations (GRE) verbal scale items versus those based on calibrations of heterogeneous items. They concluded that the differences in estimates were quite small, especially when the abilities were moderately to highly correlated. This finding supported Digvi's (1981) view that (unidimensional) IRT equating methods appear to be sufficiently robust to departures from the assumption of unidimensionality.

Cook, Dorans, Eignor, & Petersen (1985) noted that the violation of the assumption of unidimensionality observed with Scholastic Assessment Test (SAT) verbal and mathematics item responses did not seriously affect the quality of IRT true-score equating as measured by scale drift. These researchers concluded that IRT true-score equating results were acceptable with these data sets because of the presence of a dominant ability underlying the item responses to each test form, a point previously alluded to by Drasgow and Parsons (1983). More recently, Camilli, Wang, and Fesq (1992) demonstrated that the effect of multidimensionality on the IRT true-score equating of some LSAT forms was negligible. The authors did suggest, however, that the impact of multidimensionality should be investigated for those subgroups of examinees whose ability composite differs from that of the total examinee population.

Snieckus and Camilli (1993), in a simulation study, showed that the effect of a two-dimensional test structure on IRT true-score equating was insignificant as measured by scale drift, except when the means on the secondary dimension were very discrepant across simulated groups of examinees. Even in that instance, the authors questioned whether the differences were of

any practical significance.

Although these studies have provided useful information with respect to the quality of unidimensional IRT true-score equating functions in the presence of multidimensionality, they have generally tended to focus on the impact of multidimensional test content rather than investigating the interaction of both the characteristics of the items and the examinee population responding to them, as had been stressed by Lord and Novick (1968) as well as Bejar (1983).

A number of studies have attempted to assess the impact of both heterogeneous subpopulations, (i.e., violating the population invariance condition) and multidimensional test content on the quality of IRT true-score equating results (Angoff & Cowell, 1985; Cook, Eignor, & Taft, 1988; Eignor & Cook, 1991; Kingston, Leary, & Wightman, 1988; Stocking & Eignor, 1986). Angoff and Cowell (1985) noted that linear and equipercentile equating functions derived with the GRE quantitative scale tended to be relatively invariant across subgroups of examinees when the forms were homogeneous with respect to content. However, the conversions were quite discrepant with forms that showed more evidence of content heterogeneity.

Kingston, Leary, and Wightman (1988) assessed the degree of invariance of IRT true-score equating functions, derived from the Graduate Management Admissions Test (GMAT), across several subpopulations (e.g., males and females and various age groups). They concluded that the equating functions did not vary substantially across the subpopulations that were examined in their study. The authors did stress, however, that the results were obtained with subgroups that were very similar with respect to ability distribution, and test forms that were quite homogeneous with regard to content. Hence, these findings should not be generalized beyond these conditions.

Cook, Eignor, and Taft (1988) reported that different equating functions were obtained using groups of students who took the same test at different administration dates. The researchers concluded that "curricular progress" probably accounted for these differences. The implications of these findings are important for organizations that routinely administer test forms throughout the year (Eignor & Cook, 1991). Skaggs and Lissitz (1986) felt that calibrations based on samples from different parts of the United States were probably not comparable which could also potentially have an impact on IRT true-score equating. Stocking and Eignor (1986) suggested that a difference in mean ability from pretest to operational form as well as well as differences in the dimensionality of both forms might account for some of the disparities found in the conversions derived at the various stages of a test.

Skaggs and Lissitz (1988) noted that IRT true-score methods were

relatively robust to differences in examinee ability levels and suggest that multidimensional test content is probably accounting for the lack of invariance reported in previous studies. Skaggs (1990) states that the multidimensional nature of both the test and population examined is a complex issue that should be addressed more extensively.

The somewhat misunderstood relationship between multidimensionality and equating in general prompted Braun and Holland (1982) to state that it might be useful, whenever referring to an equating function, to add a qualifying phrase describing the population for which the conversion table is likely to hold. Cook and Petersen (1987) have also suggested that it might be necessary, in certain instances, to provide a description of the group for which (equated) scores can be considered to have the same meaning. It would therefore seem imperative to investigate how possible differences in the dimensional structure of a test for various subgroups of examinees might affect score equating using an IRT true-score procedure. Goldstein and Wood (1989) emphasize the importance of conducting these types of studies when they stated:

> "For various reasons to do with, say, curriculum or culture, equating relationships may vary over subpopulations, so that an overall relationship may not reflect at all accurately the relationships to be found within subgroups or subpopulations. This raises the potentially serious issue of bias and discrimination against certain subgroups. Unhappily, there appears to be little formal recognition of this in the equating literature and a lack of serious empirical study of the issue" (p. 157).

Purpose

The purpose of this study was to investigate whether differences in the dimensional structure of a form of the LSAT across selected ethnic subgroups of examinees had any impact on equating results using an IRT true-score procedure. Specifically, are there any differences in the underlying ability composite across ethnic subpopulations that might yield meaningfully different conversions for certain groups of examinees as compared to the majority group and total population equating functions?

Methods

The LSAT Equating Design

As was previously mentioned, equating enables the comparison of scores from different test forms. This procedure requires forms that are linked together through a common strand or *equating chain*. The general form of the

equating chain used with the LSAT is presented in Table 1.

Insert Table 1 about here

The equating design that is used with the LSAT is referred to as a *section pre-equating* design. As is shown in Table 1, any given LSAT form can be comprised of up to three sets of items: operational items, pre-operational items and pretest items. The reported LSAT score that an examinee receives is based solely on the operational items. Hence, every examinee is exposed to every operational item of the LSAT. The pre-operational sections are administered to examinees in order to gather statistical information that will enable us to scale the form operationally in the future. These pre-operational forms are *spiralled*. That is, each section of pre-operational items is administered to a different group of examinees. Finally, the statistical characteristics of new items are assessed through the use of pretests. Pretest items have never been administered in a previous LSAT form. As was the case with pre-operational items, pretest sections are also spiralled to various groups of examinees. Either pre-operational or pretest items are included in a *variable* section that does not contribute to the examinee's final reported score. It is referred to as a variable section because different groups of examinees are exposed to different pre-operational or pretest sections. A summary of the LSAT equating design is presented in Table 2.

Insert Table 2 about here

The three item types on the LSAT are Analytical Reasoning (AR), Logical Reasoning (LR) and Reading Comprehension (RC). Briefly, AR items measure an examinee's deductive reasoning. For example, some current AR items require an examinee to determine the proper ordering of people or objects. LR items measure an examinee's inductive reasoning. For example, some LR items require the examinee to identify flaws in a text. Finally, RC items measure the examinee's ability to read and interpret material. For some RC passages, examinees must identify the part of the stimulus that supports an inference. The LSAT form that was examined in this study contained 24 AR items, 51 LR items and 27 RC items. Hence, the raw score on this form could range from zero to 102.

Generally, when deriving a conversion table for a form of the LSAT, the first step consists of obtaining IRT parameter estimates using the marginal maximum likelihood estimation procedure implemented in the computer program BILOG (Mislevy & Bock, 1990).

Second, the item parameters obtained are scaled to the LSAT equating chain. The initial item parameters obtained for a given group of examinees cannot be compared to past populations because the metric defined by each calibration is distinct. The "arbitrariness" or indeterminancy associated with the estimation of item and ability parameters for the groups can be eliminated however by a linear transformation of item and ability parameters. The characteristic curve (CC) method developed by Stocking and Lord (1983), enables parameters on one form of a test to be scaled to another form, such that,

$$\theta^* = \lambda\theta + \kappa$$
$$b^*_i = \lambda b_i + \kappa$$
$$a^*_i = a_i \; / \; \lambda,$$

where λ and κ are scaling parameters that are selected to minimize the difference between two test characteristic curves (TCCs) obtained from two different administrations of the same form. The process of obtaining scaled parameter values that can be directly compared across groups is referred to as *item pre-equating*.

Finally, once the item and ability parameters have been placed on the same scale through item pre-equating, the ability score (θ_j estimate) for a given individual will be the same (within measurement error) irrespective of the group from which it was estimated. Therefore, if ability scores could be reported as final examinee scores, the equating process would be completed. However, two more steps are undertaken to equate the LSAT scores from a given form to the base form. First, the $P(\theta_j)$ values are summed across all 102 LSAT items for all examinees using the equation outlined in (1). This sum of $P(\theta_j)$ values is commonly referred to as an examinee's *expected true-score*. That is, the expected true-score (denoted by τ) for an examinee with ability θ_j is given by,

$$\tau = \sum_{i=1}^{n} P_i(\theta_j) \; . \qquad \textbf{(2)}$$

Unfortunately, we do not know what an examinee's true score is in reality. We only know the examinee's observed score (x). In order to overcome this situation, we treat the examinee's observed score (x) as his or her true score (τ). This approach is not without problems, however. For example, the lowest estimated true score that an examinee can achieve is equal to Σc_i (i.e., the sum of the lower asymptote item parameters) whereas the lowest observed score is in actuality, zero. Hence, the equating procedure does not function well for examinees with $x < \Sigma c_i$. Fortunately, this affects a very small proportion of examinees (in the order of 0.25% for this form of the

LSAT) and therefore is not a major impediment to the equating process. Once this process has been completed, we can equate the scores obtained on a given form to those of a *base form*. A base form is a test which serves as a comparison point in order to assess how examinees would have performed had they been exposed to this form. Hence, scores are placed onto the LSAT score scale, which ranges from 120 to 180.

Examinees

This study focused on one form of the LSAT which was administered to 45,918 examinees. Examinees who required an accommodated testing situation were excluded from the analyses. A breakdown of the examinee population is given in Figure 1.

Insert Figure 1 about here

As can be seen, the majority of the population was comprised of Caucasian examinees (34,726 or 75.6%). African-American, Asian-American and Hispanic examinees formed the three largest minority groups (respectively 3,548 or 7.7%; 3,292 or 7.2% and 1,351 or 2.9%). It is important to note that these values were self-reported by examinees. Also, the Hispanic examinee group is restricted to test takers that identified themselves as such, that is, it does not include test takers who describe themselves as being Puerto-Rican or Mexican-American. The analyses in this study were centered upon the majority (Caucasian) group as well as two minority groups, that is African-American and Hispanic examinees.

A description of the dimensionality assessment procedures

The first set of analyses was centered on assessing the dimensionality of the LSAT form with the Caucasian, African-American and Hispanic subgroups. Dimensionality was assessed using two procedures: Stout's essential dimensionality procedure and T statistic (Stout, 1987; 1990), as well as an approximate χ^2 statistic based on McDonald's nonlinear factor analytic (NLFA) model (De Champlain, 1992; Gessaroli & De Champlain, 1994; McDonald, 1967; 1982).

Stout's essential dimensionality procedure

Stout proposed a nonparametric procedure that is based on his notions of *essential independence* and *essential dimensionality* (Nandakumar, 1991; 1993; Stout, 1987; 1990). Essential dimensionality corresponds to the number of latent traits that are required to satisfy the assumption of essential

independence, that is, a mean absolute residual covariance value that tends towards zero after ability has been partialled out,

$$\frac{1}{N(N-1)} \sum_{1 \le i \le j \le N} |COV(U_i, U_j | \theta)| \approx 0 . \qquad (3)$$

A test (U_1, U_2, ..., U_N) is said to be essentially unidimensional if for all subsets (U_1, U_2, ..., U_M) of length M(<N) and all values of Y_p,

$$\frac{1}{M(M-1)} \sum_{1 \le i \le j \le N} |COV(U_i, U_j | Y_p)| \approx 0 . \qquad (4)$$

where Y_p is the proportion correct score on the longer subtest and (U_1, U_2, ..., U_M) are shorter subtests with length n = N-M. Stout (1987; 1990) proposed the *T* statistic to test the assumption of essential unidimensionality. The steps involved in the calculation of the *T* statistic are outlined in Stout (1987) and Nandakumar (1991). In a series of studies carried out by Stout and his colleagues, the *T* statistic was found to be quite accurate in correctly determining essential unidimensionality or departure from the assumption with multidimensional data sets (Junker & Stout, 1991; Nandakumar, 1994) except when the test contained few items (less than 25) and the sample sizes were small (less than 750 examinees) (De Champlain, 1992; Nandakumar, 1987).

Nonlinear factor analysis

Another approach that is gaining popularity for the assessment of dimensionality is the one that treats IRT as a special case of NLFA. Several researchers have shown that common IRT models are a special case of a more general NLFA model and that the functions are mathematically equivalent (Bartholomew, 1983; Goldstein & Wood, 1989; McDonald, 1967; 1991; Takane & De Leeuw, 1987). Based on this IRT-nonlinear factor analytic (NLFA) relationship, some researchers have suggested that the most suitable method of assessing dimensionality should be based on the analysis of the residual covariance matrix obtained after fitting a *k*-factor NLFA model (Gessaroli, in press; Goldstein, 1980; Goldstein & Wood, 1989; McDonald, 1981; in press). An approximate χ^2 statistic, based on McDonald's NLFA model, was investigated by De Champlain (1992) and Gessaroli and De Champlain (1994) as a potentially useful procedure for assessing dimensionality. The approximate χ^2 was originally proposed by Bartlett (1950) and outlined by Steiger (1980a; 1980b). The approximate χ^2 statistic tests the null hypothesis that the off-diagonal elements of a residual correlation matrix are equal to zero after fitting an *m*-factor NLFA model. The statistics are based on the estimation of parameters for an *m*-factor model using the NLFA approach outlined by McDonald (1967) and

implemented by Fraser and McDonald (1988) in the computer program NOHARM. The approximate χ^2 statistic can be defined as,

$$\chi^2 = (N-3) \sum_{i=1}^{p} \sum_{j=1}^{i-1} z_{ij}^{2(r)},$$

where $z_{ij}^{2(r)}$ is the square of the Fisher z corresponding to the residual correlation between items i and j, $(i, j = 1 \ldots p)$ and N is the number of examinees in the sample. This statistic is distributed approximately as a central χ^2 with df = $.5k\ (k - 1) - t$ where k is equal to the number of items and t is the total number of independent parameters estimated. The specific computational steps for this approximate chi-square statistic are given in Appendix A. The performance of the approximate χ^2 statistic was assessed with simulated unidimensional and multidimensional data sets that varied according to test length, sample size, item parameter values as well the number of items defining each latent trait (De Champlain, 1992; Gessaroli & De Champlain, 1994). With unidimensional data sets, the empirical Type I error rates tended to be lower than the nominal α for the shorter test lengths examined but increased to values close to expected α probabilities for the longer tests. Also, the empirical Type I error rates obtained for the approximate χ^2 statistic were not affected by nonnormal ability distributions (De Champlain & Tang, 1993). With multidimensional data sets, rejection rates based on the approximate χ^2 statistic were generally high, even in some instances with data sets containing as few as 15 items and 500 examinees, which was not the case with Stout's T statistic (De Champlain, 1992; Gessaroli & De Champlain, 1994).

A description of the dimensionality assessment analyses

Initially, the fit of a unidimensional model was assessed using the two above defined procedures. Stout's T statistic was computed for the Caucasian, African-American and Hispanic LSAT data sets using the computer program DIMTEST (Stout, Junker, Nandakumar, Chang, & Steidinger, 1991). It was not possible to analyze all (102) items of the LSAT form due to program restrictions. Hence, the last two (RC) items were dropped, (i.e., the first 100 items were subjected to DIMTEST). Also, the approximate χ^2 statistic proposed by Gessaroli and De Champlain (1994) was calculated for the same three data sets with the computer program CHIDIM (De Champlain & Tang, 1994), after fitting a one-factor model using NOHARM (Fraser & McDonald, 1988).

The fit of more complex models (e.g., two- and three-factor models) was also assessed with NOHARM and the approximate χ^2 statistic, as computed by

CHIDIM (De Champlain & Tang, 1994). Past research has shown that there appear to be two correlated abilities underlying the item responses to recent forms of the LSAT (Ackerman, 1994; Camilli, Wang, & Fesq, 1992; Roussos & Stout, 1994), including the form that was the focus of this study (De Champlain, 1994a). More precisely, the first factor corresponds to AR whereas the second reflects a combination of both LR and RC. The appropriateness of this model was ascertained for the three ethnic groups by fitting a confirmatory two-factor model to the matrices of item responses and by calculating the approximate χ^2 statistic to see if the sum of the squared residuals differed significantly from zero. Finally, the fit of a three-factor model, specifying AR, LR and RC as separate dimensions, was examined for these same three groups of examinees, again with the approximate χ^2 statistic.

Given that χ^2-distributed statistics often suffer from an inflated Type I error rate with large sample sizes (Marsh, Balla, & McDonald, 1988), random samples of Caucasian and African-American examinees were selected for all dimensionality analyses. Specifically, a random sample of 1351 Caucasian examinees and African-American examinees was selected for these analyses in order to match the sample size of the Hispanic subgroup. Both procedures have been shown to be generally accurate with respect to correctly identifying the unidimensional or multidimensional nature of item response matrices with these sample sizes (De Champlain, 1992; Gessaroli & De Champlain, 1994; Nandakumar, 1994).

Equating analyses

The second set of analyses entailed deriving and comparing separate conversion functions for the Caucasian, African-American and Hispanic subgroups to see whether any noticeable discrepancies, possibly attributable to differences in the dimensional structure of the ethnic group item response matrices, might exist. Specifically, the equating functions for the three groups were compared to the total population conversion line, that is, the one actually used in the past for score reporting. In addition, the African-American and Hispanic equating functions were plotted against the majority group (i.e. Caucasian) conversion line to see whether any noticeable discrepancies occurred. In order to obtain these equating functions, three steps were followed.

First, separate IRT parameter estimates were obtained for each group of examinees using the computer program BILOG (Mislevy & Bock, 1990). Default BILOG program values were used for all analyses.

Second, the item parameters obtained were scaled to the LSAT equating chain using the CC method. For this study, the IRT parameters obtained for

each ethnic group were scaled to the LR pre-operational form parameters (51 items) obtained from the total population using the SCALE program (McKinley, 1993). Past research has shown that the LR items on the LSAT showed the most evidence of unidimensionality across forms and random samples (De Champlain, 1994b; Roussos & Stout, 1994). Also, past studies have demonstrated that 40 common items generally result in an adequate scaling when using a characteristic curve procedure (Wingersky, Cook, & Eignor, 1986). The IRT parameters obtained for the total examinee population were scaled to pre-operational parameter values. That is, all 102 item were included in the scaling analysis.

Once the item and ability parameters were placed on the same scale through item pre-equating, the $P(\theta_j)$ values were summed across all 102 LSAT items for all examinees in order to obtain their estimated true scores. As was mentioned previously, the true-score conversion table obtained for each group was then applied to their respective raw scores given that true-scores are unknown in reality (Lord, 1980). Once this process was completed, scores for all examinees were equated to those of a base form. This was accomplished by using the EQUATE program (McKinley, 1993).

Results

Descriptive statistics

LSAT raw score descriptive statistics obtained with the three groups of examinees and the total population are presented in Table 3. Also, their respective distributions are plotted in Figure 2.

Insert Table 3 and Figure 2 about here

The differences that were obtained between the groups with respect to both mean raw score and frequency distribution are very similar to those reported with other national testing programs, most notably the GRE General test (Briel, O'Neill, & Scheuneman, 1993).

Dimensionality analyses

Fit of a unidimensional model

Initially, the fit of a unidimensional model to the item response matrices of the Caucasian, African-American and Hispanic subgroups was ascertained using Stout's T statistic and the approximate χ^2 statistic obtained after fitting a (one-factor) NLFA model. These results are summarized

in Table 4.

Insert Table 4 about here

Irrespective of the procedure employed, there is ample evidence to confirm the multidimensional nature of this form of the LSAT with all three subgroups of examinees. Stout's *T* statistic was statistically significant for all data sets. In addition, an inspection of the approximate χ^2 statistic values reveals that the sum of the squared residual correlations differed significantly from zero after fitting a one-factor NOHARM model, again clearly confirming the multidimensional nature of the data set for the three ethnic groups.

Fit of the two- and three-factor models

The fit of a two- and three-factor model to the item response matrices of the Caucasian, African-American and Hispanic subgroups was also investigated using NLFA and the approximate χ^2 statistic. The approximate χ^2 statistic values are shown in Table 5.

Insert Table 5 about here

Results indicate that the two-factor confirmatory NLFA model fit the item response matrices of both the Caucasian and African-American examinees. In other words, the residual correlations did not differ significantly from zero after fitting the two-dimensional model. These results support those of Ackerman (1994), Camilli, Wang, and Fesq (1992), De Champlain (1994a) as well as Roussos and Stout (1994) who had suggested that two dominant abilities were needed to correctly answer the items on the LSAT, that is, AR and a combination of both LR and RC. However, this two-factor model did not adequately account for the item responses of the Hispanic subgroup of examinees, $\chi^2(4946, N= 1351) = 5875.798$, $p<.000001$. In addition, the fit of a three-factor model, specifying AR, LR and RC as separate dimensions, was still inadequate in accounting for the item responses of Hispanic examinees, $\chi^2(4842, N = 1351) = 5663.276$, $p<.000001$.

Equating analyses

Total population and African-American equating comparisons

The raw- to unrounded scaled-score conversion functions derived for the African-American subgroup and the total examinee population are plotted in

Figure 3.

Insert Figure 3 about here

The differences obtained between the two conversions were very small. The mean absolute difference between the two score conversions, weighted by the number of African-American examinees at each corresponding raw score point, was equal to 0.50 with a standard deviation of 3.08. A plot of the differences in the equated scores for the two groups across the raw-score metric is provided in Figure 4.

Insert Figure 4 about here

Again, this plot shows that the differences between the two conversions are of little practical significance and are well within one conditional standard error of measurement of score differences (CSEM DIFF) (Dorans, 1994) across the entire raw-score metric of the LSAT. In other words, if we were to compute the differences between scores for all pairs of total examinee population and African-American examinees of the same true ability, 68% of these differences would fall within one pair of CSEM values at each score point. None of the differences plotted in Figure 4 were beyond these ranges. Not surprisingly, the largest discrepancies occurred at the lower end of the scale where the paucity of scores contributes to a poorer fit of the model and consequently a larger amount of measurement error.

Caucasian and African-American equating comparisons

The raw- to unrounded scaled-score conversion functions derived for the African-American and Caucasian subgroups are plotted in Figure 5.

Insert Figure 5 about here

The differences obtained between the two conversions were also very small. The mean absolute difference between the two conversions, weighted by the number of African-American examinees at each corresponding raw score point, was equal to 0.29 with a standard deviation of 1.47. A plot of the differences in the equated scores for the two groups across the raw-score metric is provided in Figure 6.

Insert Figure 6 about here

Again, this plot shows that the differences between the two conversions are of little practical significance and are well within one conditional standard error of measurement of score differences (CSEM DIFF) (Dorans, 1994) across the entire raw-score metric of the LSAT. None of the residuals plotted in Figure 6 were beyond their respective CSEM DIFF values. Once more, the largest discrepancies occurred at the lower end of the scale where the scarcity of scores yields a poorer fit of the model and consequently a larger amount of measurement error. The true-score conversions obtained from 10 randomly selected samples of Caucasian examinees (N=3,000) were also plotted against the African-American equating function in order to determine whether there was any noticeable pattern in the residuals. These plots are shown in Figure 7.

Insert Figure 7 about here

The results were quite similar to those obtained with the total Caucasian population in that the differences across most of the scaled score range were negligible for the 10 comparisons. Also, the largest residuals occurred at the lower end of the scale which again can be attributed to the larger amount of error concentrated in the segment of the scale that contains very few scores.

Given the differences in raw-score distributions previously noted between the two groups, it was also of interest to examine whether matching Caucasian examinees on the basis of the African-American raw-score frequency distribution might lead to even smaller discrepancies between the two functions. However, it was not possible to obtain a perfect match at the lower end of the raw-score metric. The raw-score frequency distributions for the matched Caucasian subgroup and the African-American population are shown in Figure 8.

Insert Figure 8 about here

As shown in Figure 8, with the exception of the very low end of the raw-score scale, the two distributions were identical. A plot of both unrounded score conversions as well as the residuals between the two equating functions are presented in Figures 9 and 10.

Insert Figure 9 and Figure 10 about here

The differences obtained between the two conversions were slightly larger when matching on number-right score but still inconsequential. The mean absolute difference between the two conversions, weighted by the number of African-American examinees at each corresponding raw score point, was equal to 0.40 with a standard deviation of 3.88. Matching on the basis of raw-score had a slight impact on the differences between the two conversion lines. Matching appears to have increased the differences between the two conversions at the very low end of the raw-score metric. It's important to stress, however, that these differences were still within one CSEM DIFF across the entire raw-score metric. Conversely, matching did reduce the discrepancies between both conversions in the middle-range of the distributions, where most scores are concentrated. However, the differences at the upper end of the raw-score scale were slightly higher that previously observed with the (unmatched) Caucasian group. It is important to note that the highest raw score for the African-American group (and consequently, the matched Caucasian group) was 95. Therefore, the differences noted at the extreme end of the scale can perhaps partially be imputed to the interpolation process employed by the EQUATE program in the absence of information (i.e., scores), rather than to any meaningful disparities.

Total population and Hispanic equating comparisons

The raw- to unrounded scaled-score conversion functions derived for the Hispanic subgroup and the total examinee population are plotted in Figure 11.

Insert Figure 11 about here

The differences obtained between the two conversions were very small. The mean absolute difference between the two conversions, weighted by the number of Hispanic examinees at each corresponding raw score point, was equal to 0.41 with a standard deviation of 2.63. A plot of the differences in the equated scores for the two groups across the raw-score metric is provided in Figure 12.

Insert Figure 12 about here

Again, this plot clearly shows that the differences between the two conversions are of little practical significance and are well within one

conditional standard error of measurement of score differences (CSEM DIFF) (Dorans, 1994) across the entire raw-score metric of the LSAT. As was the case for previous comparisons, none of the differences plotted in Figure 12 were beyond these ranges. The largest discrepancies also occurred at the lower end of the scale due to the paucity of scores and consequently the larger amount of measurement error in that segment of the scale.

Caucasian and Hispanic equating comparisons

The raw- to unrounded scaled-score conversion functions derived for the Hispanic and Caucasian subgroups are plotted in Figure 13.

Insert Figure 13 about here

Once more, the differences obtained between the two conversions were virtually nil. The mean absolute difference, weighted by the number of Hispanic examinees at each raw score point, was equal to 0.11 with a standard deviation of 0.55. A plot of the differences in the equated scores for the two groups across the raw-score metric is provided in Figure 14.

Insert Figure 14 about here

It is clear from this plot that the differences between the two conversions are of little practical significance and are again well within one CSEM DIFF. As was the case with the previous comparisons, none of the differences plotted in Figure 14 were beyond this range. Also, the largest differences occurred at the two extremes of the raw-score scale where few examinee scores are located. Residual plots comparing the equating functions derived from the 10 random sample of Caucasian examinees (N=3,000) against the Hispanic population conversion are shown in Figure 15.

Insert Figure 15 about here

Once more, the differences between the pairs of conversions were negligible and generally tended to be concentrated at the lower end of the raw-score metric.

Discussion

IRT true-score equating methods assume that the construct underlying the items of the forms to be equated is unidimensional in nature. This assumption

of the model must be met in order to benefit from the many advantages of IRT-based procedures, including population invariance. Simply stated, IRT equating functions should theoretically be independent of the groups from which they were derived, assuming the postulates of the models hold.

Previous studies that had assessed the relationship between multidimensionality and IRT true-score equating results generally concluded that this procedure was quite robust to departures from the assumption of unidimensionality (Bogan & Yen, 1983; Camilli, Wang, & Fesq, 1992; Cook & Douglass, 1982; Cook, Dorans, Eignor, & Petersen, 1985; Dorans & Kingston, 1985; Kolen & Whitney, 1982; Modu, 1982; Snieckus & Camilli, 1993; Stocking & Eignor, 1986; Wang, 1985; Yen, 1984). However, these studies focused on multidimensionality as a sole property of the content, rather than as an interaction of both examinee population and the characteristics of a set of items. Studies that did examine the interaction of both multidimensional test content and heterogeneous population concluded that this combination of factors might impact on (unidimensional) IRT true-score equating (Angoff & Cowell, 1985; Cook, Eignor, & Taft, 1988; Eignor & Cook, 1991; Kingston, Leary, & Wightman, 1988; Stocking & Eignor, 1986). The purpose of this study was therefore to assess the dimensionality of one form of the LSAT with three ethnic groups of examinees and to investigate whether differences in the ability composite have any noticeable impact on IRT true-score equating results for these subgroups. Specifically, the conversion lines estimated for African-American and Hispanic examinees were compared to the equating functions derived from the majority (Caucasian) group as well as the total examinee population to see whether notable differences existed between the functions.

Results obtained with respect to the dimensionality of the LSAT with the three ethnic groups showed that a two-dimensional model, previously reported with the total population, adequately accounted for the item responses of both Caucasian and African-American examinees. Specifically, an AR as well as a LR + RC ability appear to underlie the item response matrices of both groups of examinees. However, the degree of misfit of this model was quite large for Hispanic examinees. In fact, a three-factor model, specifying AR, LR and RC as separate dimensions, was still inadequate with regard to explaining Hispanic examinee item responses. Note that the makeup of the Hispanic examinee group might be quite varied and contain distinct subgroups with respect to their LSAT ability composite. For example, it is possible that the Hispanic population is comprised of one group of students who are quite fluent in English and a second group for whom English is a second language. This would have to be investigated more thoroughly before making any definite conclusions

regarding the factor structure of the LSAT for these examinees.

Equating results generally indicated that the differences between the conversion lines obtained for the three ethnic groups and the total examinee population were negligible, especially throughout the segment of the scale that contained most scores. In this sense, these findings confirm those reported in previous studies that examined the degree of invariance of IRT true-score equating functions across subgroups of examinees (Angoff & Cowell, 1985; Kingston, Leary, & Wightman, 1988).

However, it is important to note that the results obtained in this study also suggest that differences in the underlying ability composite between groups yielded conversions that did not differ substantially. The differences that were noted occurred primarily at the lower end of the raw-score metric which is to be expected given the small number of scores concentrated in that segment of the scale and hence the larger amount of measurement error Dorans and Kingston (1985) as well as Petersen, Cook and Stocking (1983) similarly attributed larger residuals in the tails of the scale to the paucity of observations contributing to a poorer fit of the model for these scores. Also, it is known that the IRT true-score equating procedure does not function particularly well for examinees' whose observed score is lower than their expected true score.

Therefore, the discrepancies noted at the lower end of the scale are probably attributable to the shortcomings of the model rather than ability composite divergences among groups. Also, the largest residuals obtained when comparing the minority group conversion lines to either the Caucasian or total population equating functions were well within one conditional standard error of measurement for score differences which again would indicate that the variations are of little practical significance. It is particularly surprising to see that the differences obtained between the Hispanic conversion line and the total population as well as Caucasian equating functions were so insignificant given their distinctly different underlying ability composite. Perhaps, the two dimensions that account for the Caucasian and African-American item response matrices also predominantly underlie Hispanic examinee item responses, in addition to other minor abilities. Again, a more thorough investigation would seem necessary before drawing any definite conclusions as to why the equating functions were invariant across ethnic subgroups even in the presence of different ability composites.

Matching Caucasian examinees on the basis of the African-American raw-score frequency distribution tended to increase the disparities between the equating functions at the extremes, hence contributing to a slightly larger mean absolute residual value. However, the discrepancies between the two

conversion lines in the middle of the scale were smaller. These findings support those of Cook, Eignor, and Schmitt (1990) as well as Kolen (1990) who stated that matching generally did not contribute to a more accurate equating. Perhaps, as Skaggs (1990) pointed out, the complex multidimensional relationship that exists between test form and population might account for the questionable benefits that are sometimes reported in the literature with respect to the effect of matching on equating.

Regardless, the results obtained in this study suggest that African-American and Hispanic conversion lines are, from a practical standpoint, equivalent to the equating function of the majority Caucasian group as well as to the one derived from the total examinee population. In other words, the current practice of applying a conversion function obtained from the total population to all examinees, irrespective of ethnicity, does not penalize minority examinees, as evidenced by the residual plots produced in this investigation. Also, the equating residual plots between minority and majority group examinees did not seem to conform to any apparent pattern, as displayed in the two sets set of figures comparing the random Caucasian samples and minority group equating functions.

Limitations of the study

The results obtained in this study are based on only one form of the LSAT and two minority groups. Hence, the negligible effect of multidimensionality on the equating functions of African-American and Hispanic examinees cannot be generalized to other ethnic subgroups or LSAT forms without further investigation.

Also, the assessment of the dimensionality of the LSAT form was based upon only two procedures. Although the literature has shown that Stout's T statistic and NLFA are two very promising methods for the assessment of dimensionality, it might be useful to also apply other techniques, for example, full-information factor analysis (Bock & Aitkin, 1981) as implemented in TESTFACT (Wilson, Wood, & Gibbons, 1987).

Suggestions for future research

It is hoped that the results reported in this study will foster future research with respect to the relationship between multidimensionality, population invariance and IRT true-score equating results.

This study should be replicated over several ethnic groups and forms of not only the LSAT but also other national testing programs to see if similar results are obtained.

It might also be of interest to attempt to examine the relationship

between multidimensionality, population invariance and equating results using several models, for example, linear, equipercentile and IRT observed-score procedures, to ascertain how each method is affected.

Also, the effect of other socio-demographic variables, e.g., degree of English fluency, with respect to IRT true-score equating results could be modeled in order to better understand how robust this procedure is with heterogeneous examinee populations.

Examining the relationship among multidimensionality, population invariance and equating is especially relevant within a computer adaptive testing (CAT) framework. Currently, at each administration, all examinees are exposed to a single form of the LSAT. In that sense, the current paper-and-pencil LSAT administration represents a highly constrained environment. However, within a CAT perspective, a different LSAT form is essentially "tailored" to each examinee. Hence, the number of factors that must be taken into consideration with a CAT far outweighs what is presently encountered with the paper-and-pencil form of the test. Although the results obtained in this study are encouraging, the analyses should be replicated over several subsets of items in order to better understand how the equating of CAT forms might be affected by multidimensionality and population invariance.

Hopefully, the results obtained in these and other studies will provide valuable guidelines to practitioners regarding the degree of robustness of IRT- and classically-based equating procedures to violations of unidimensionality and the degree of invariance to be expected with heterogeneous subgroups of examinees.

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Table 1

Hypothetical LSAT Equating Chain

Operational forms	Pre-operational forms	Pretests
Base Form (1)	2	a,b,c,d
2	3	e,f,g,h
3	4	i,j,k,l
4	5	m,n,o,p
5	6	q,r,s,t

Table 2

LSAT Section Pre-Equating Design

	Operational sections				Variable section[1]				
Sample	AR[2]	LA[3]	LB	RC[4]	V_1	V_2	V_3	.	V_z
S_1	✓	✓	✓	✓	✓				
S_2	✓	✓	✓	✓		✓			
S_3	✓	✓	✓	✓			✓		
.									
S_z	✓	✓	✓	✓					✓

[1] A variable section can correspond to either a pre-operational or pretest LSAT section.

[2] AR = Analytical Reasoning

[3] LA+LB = Logical Reasoning

[4] RC = Reading Comprehension

Table 3

LSAT Raw Score Descriptive Statistics by Ethnicity

Ethnic group	Mean	Standard deviation	Skewness	Kurtosis
Total (N=45,918)	59.291	15.729	-0.087	-0.343
Caucasians (N=34,726)	61.384	14.817	-0.090	-0.228
African-Americans (N=3,548)	45.198	13.994	0.441	0.020
Hispanics (N=1,351)	52.882	15.713	0.196	-0.489

Table 4

Assessing the Fit of a Unidimensional Model

Ethnic group	Stout's T statistic	Approximate χ^2 and p value 1-F NOHARM model
Caucasians	2.90, $p<.001$	$\chi^2 = 9,239.50$; $p<.001$
African-Americans	2.71, $p<.003$	$\chi^2 = 8,288.29$; $p<.001$
Hispanics	3.30, $p<.001$	$\chi^2 = 13,625.44$; $p<.001$

Table 5

Assessing the Fit of Two- and Three-Factor Models

	2-factor NOHARM model	3-factor NOHARM model
Ethnic group	Approximate χ^2 and p values	Approximate χ^2 and p values
Caucasians	χ^2 = 4,890.76; p<.074	χ^2 = 4,655.36; p<.163
African-Americans	χ^2 = 4,542.291; p<.985	χ^2 = 4,427.44; p<.990
Hispanics	χ^2 = 5,875.80; p<.001	χ^2 = 5,663.28; p<.001

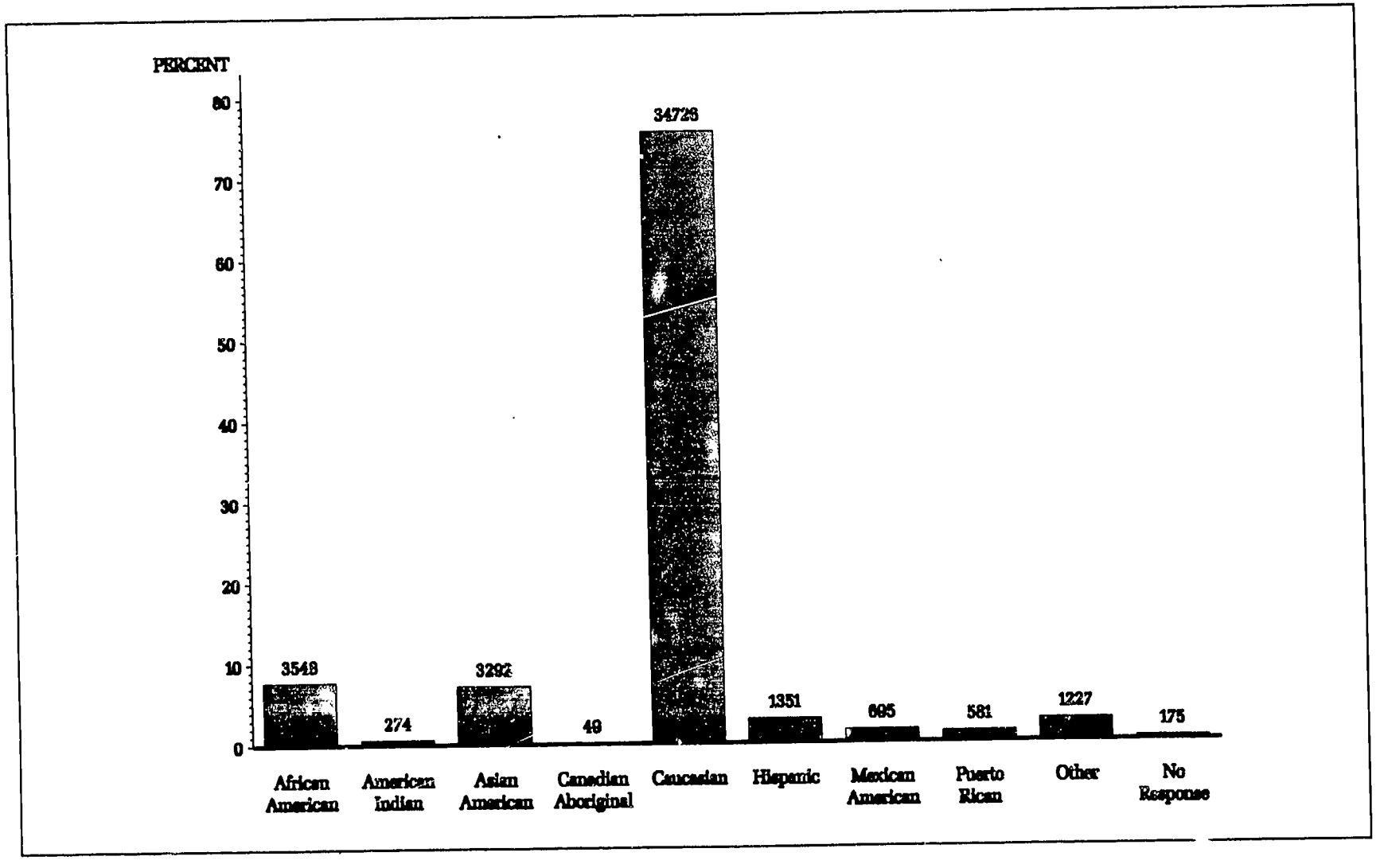

FIGURE 1. LSAT Ethnicity Frequency Distributions

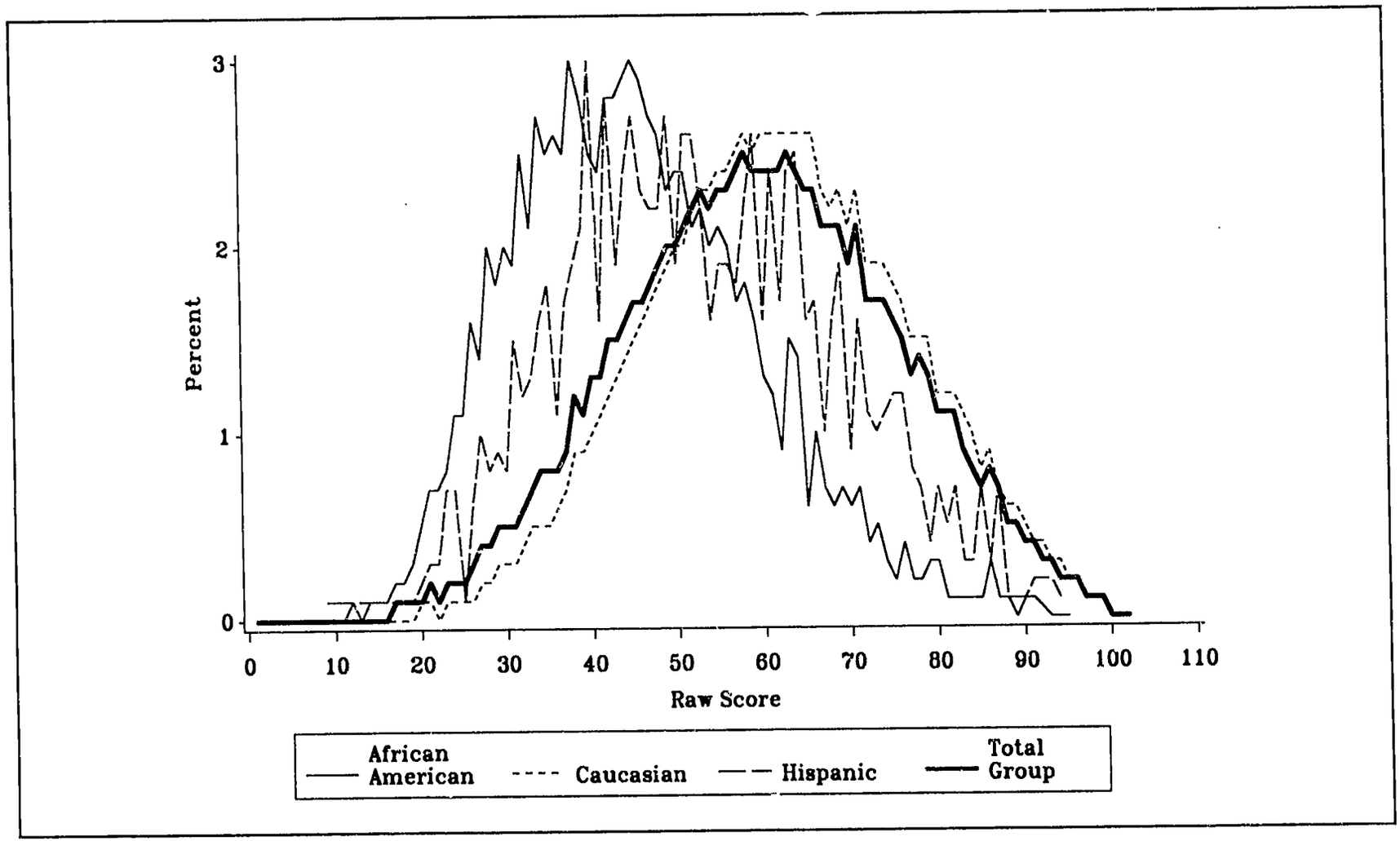

FIGURE 2. LSAT Raw Score Percentage Distributions for African-American, Caucasian, Hispanic, and Total Group Examinees

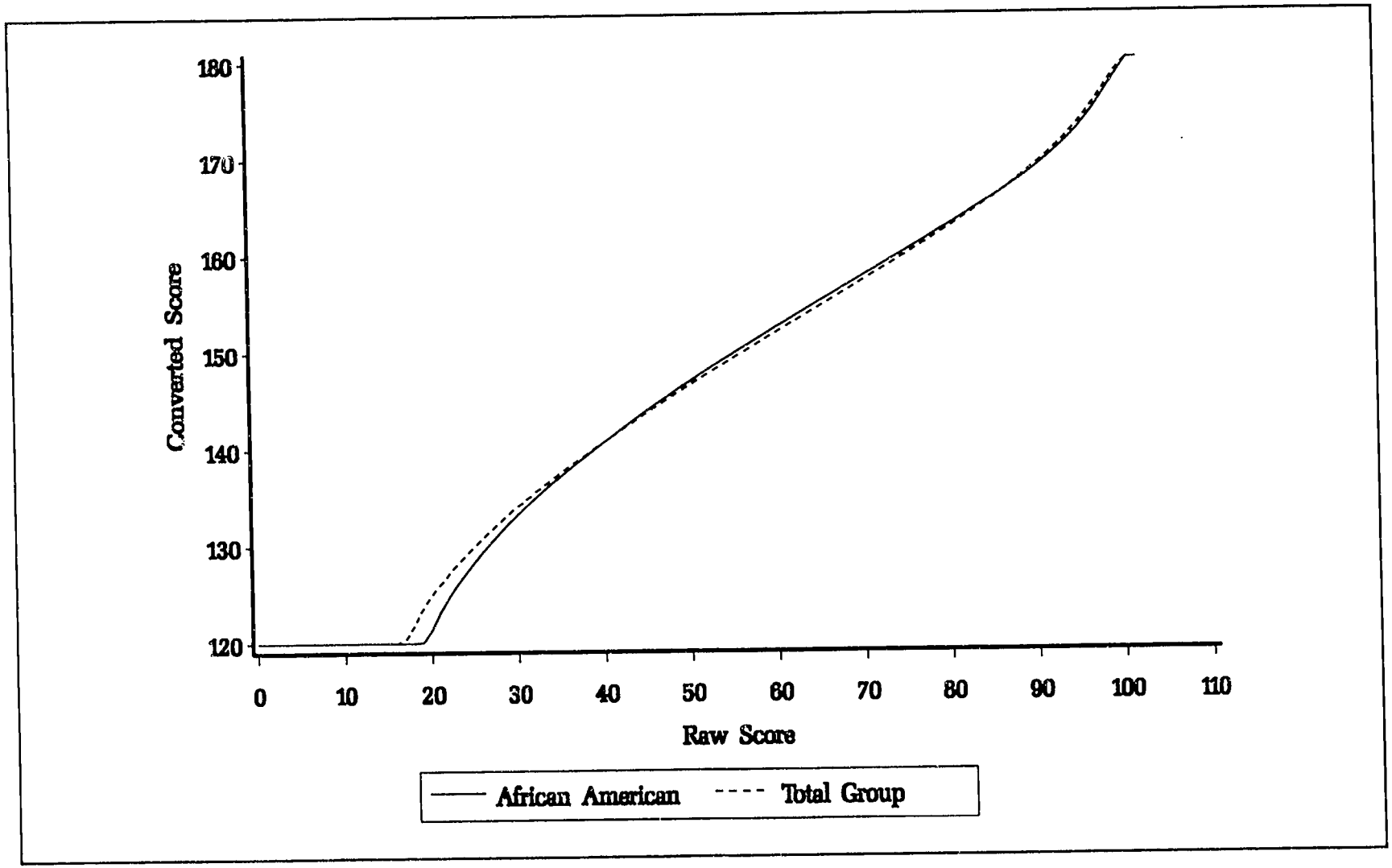

FIGURE 3. LSAT Total Group and African-American Raw to Scaled Score Conversion Lines

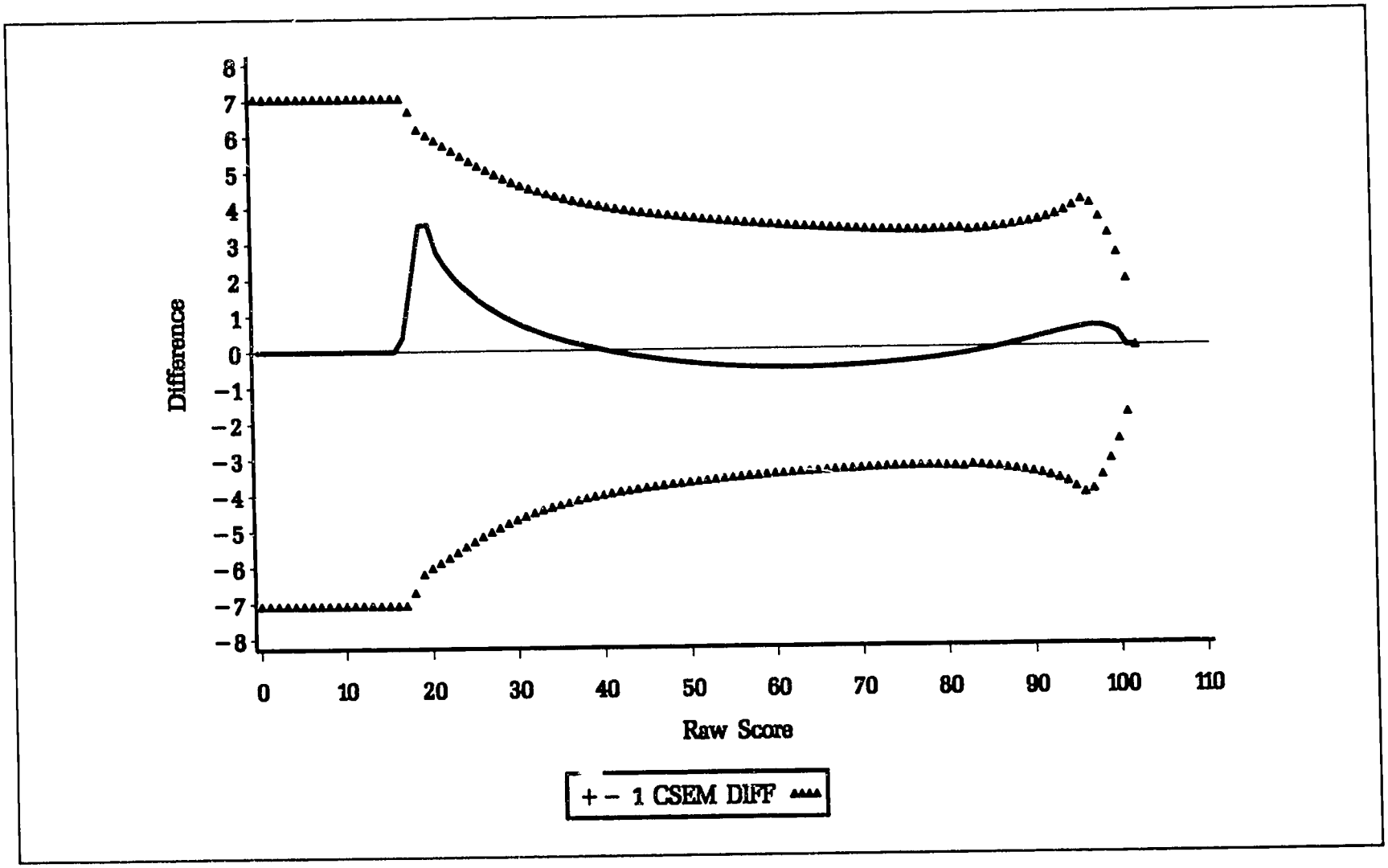

FIGURE 4. Total Group – African-American Equating Residual Plot

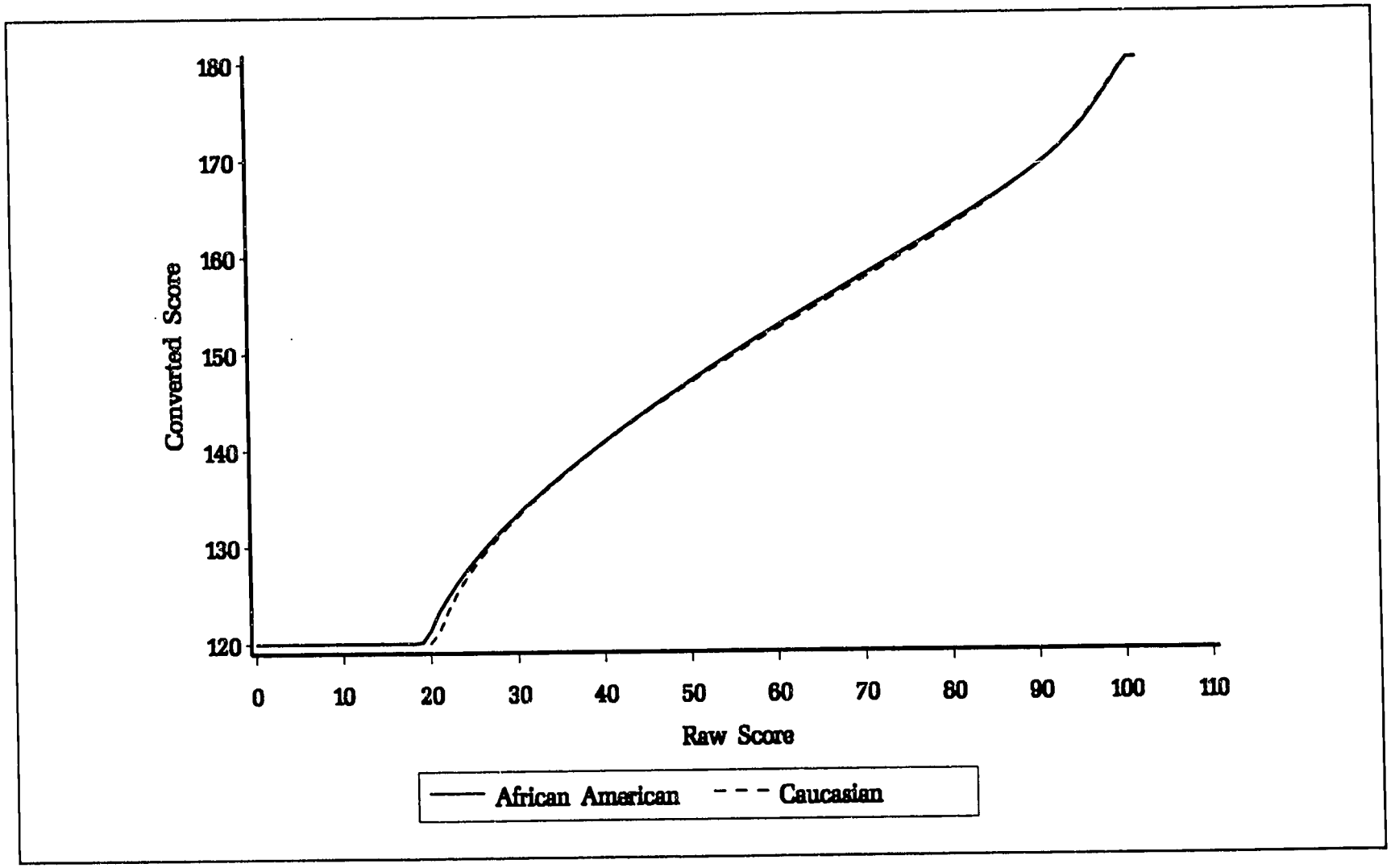

FIGURE 5. LSAT African-American and Caucasian Raw to Scaled Score Conversion Lines

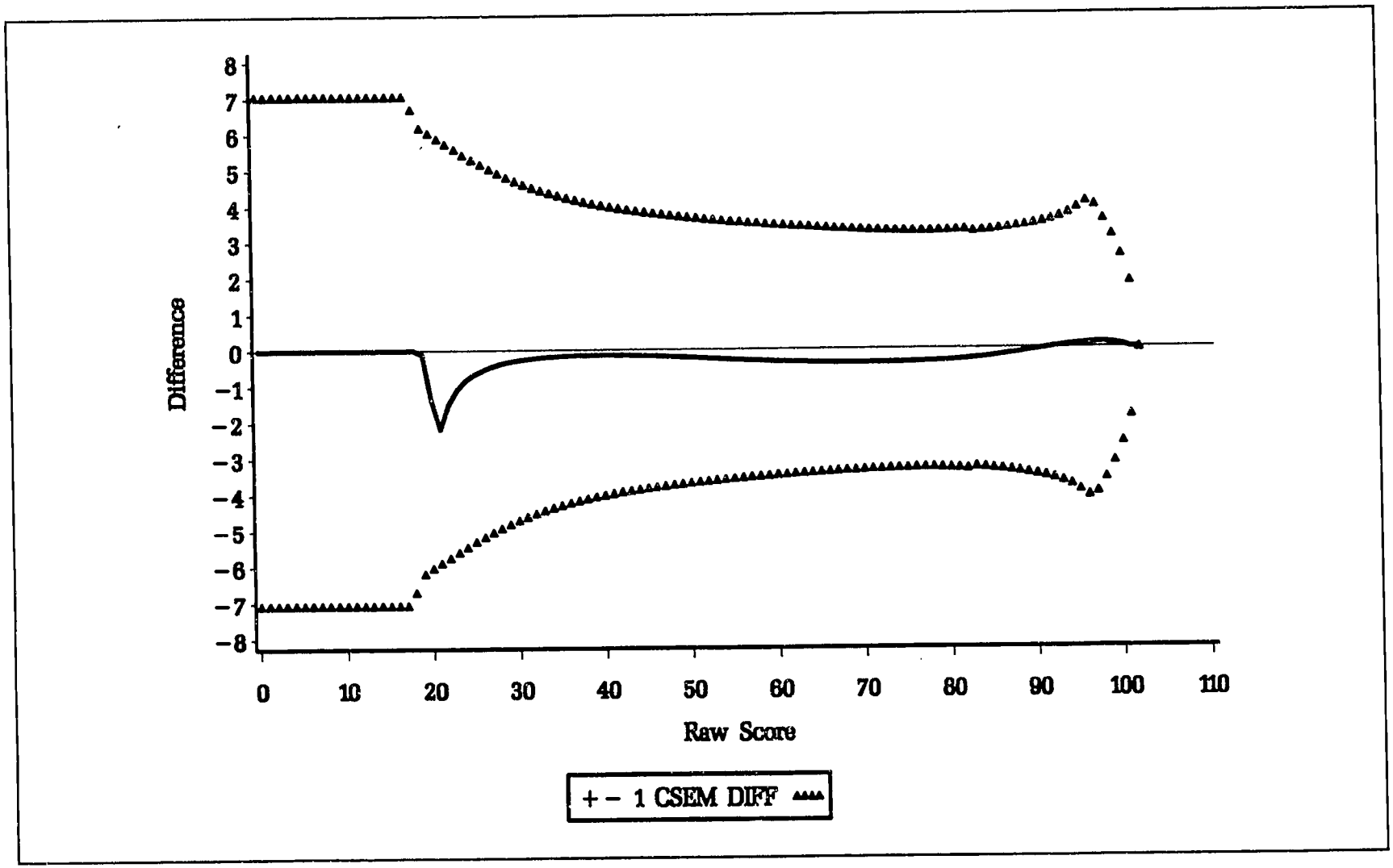

FIGURE 6. Caucasian–African-American Equating Residual Plot

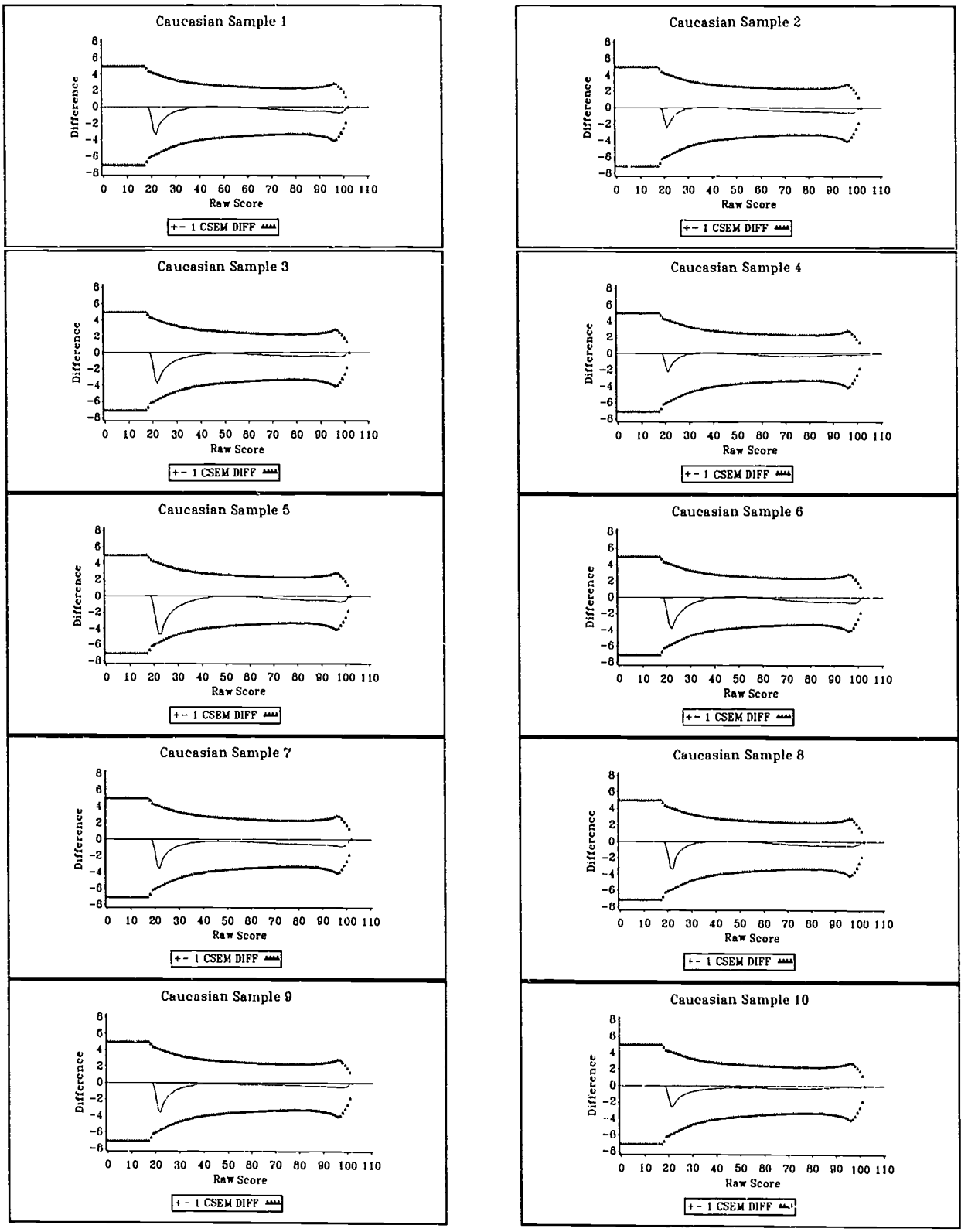

FIGURE 7. Random Sample Caucasian-African-American Equating Residual Plots

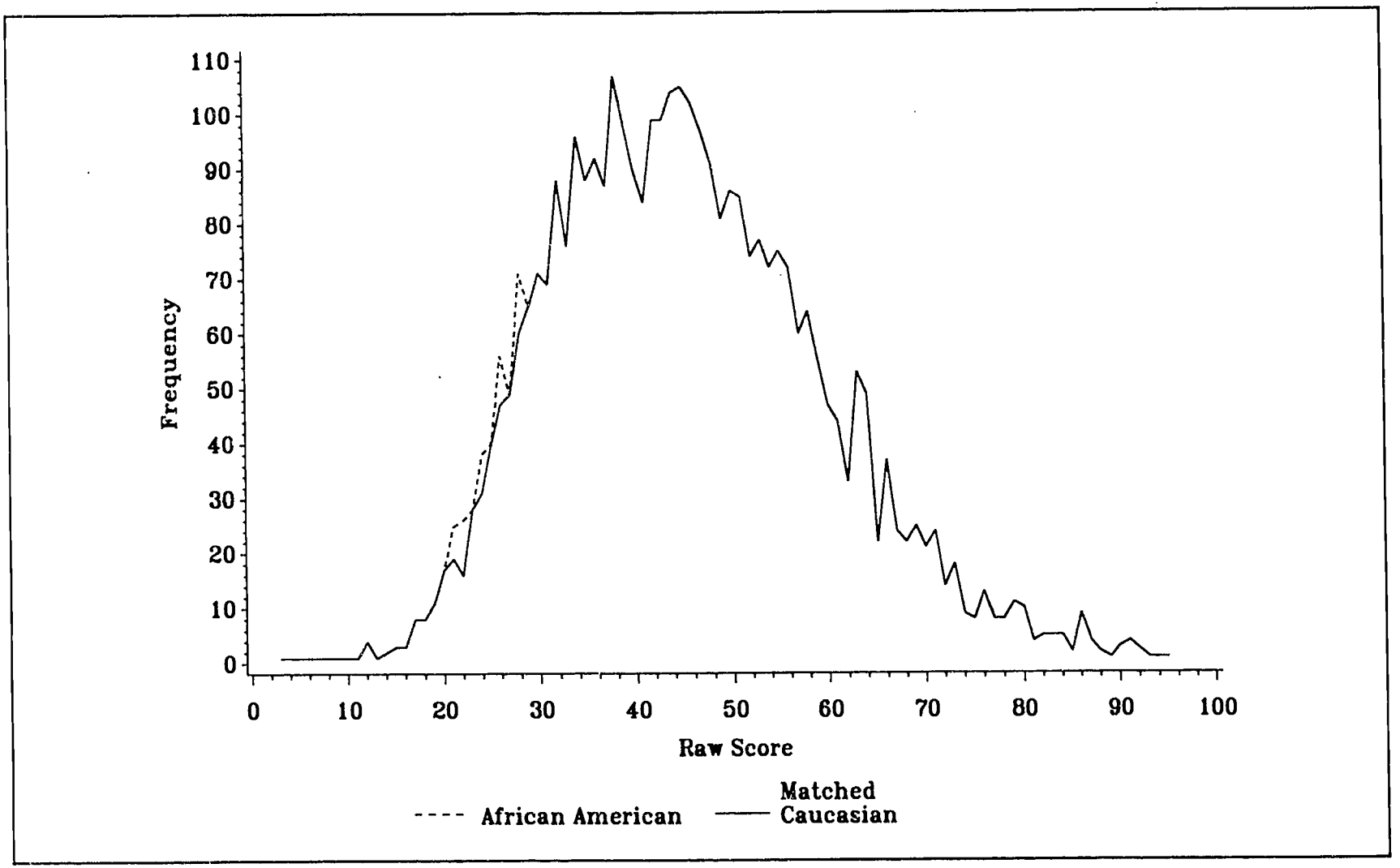

FIGURE 8. LSAT Raw Score Frequency Distributions for African-American and Matched Caucasian Sample Examinees

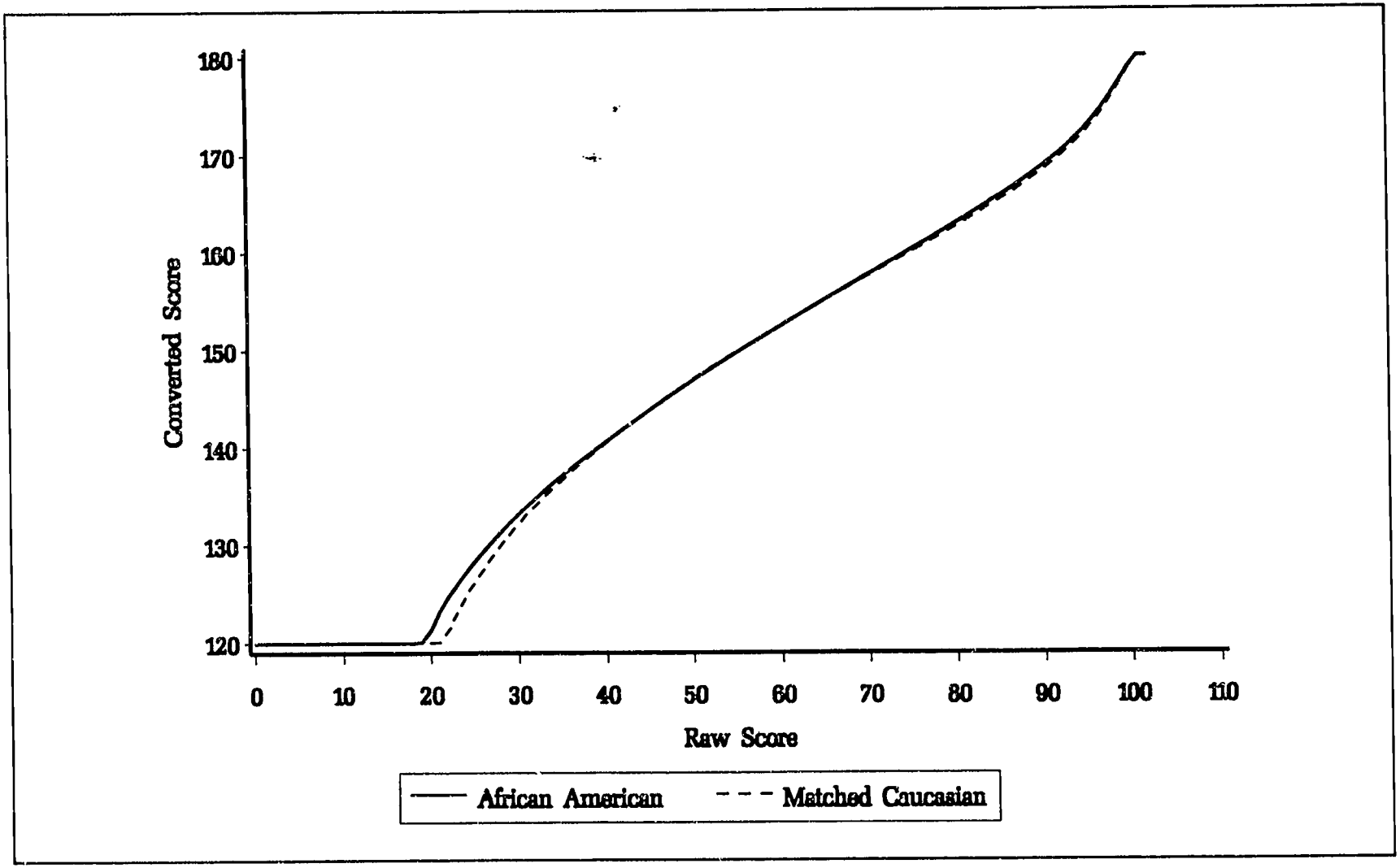

FIGURE 9. LSAT African-American and Matched Caucasian Sample – Raw to Scaled Score Conversion Lines

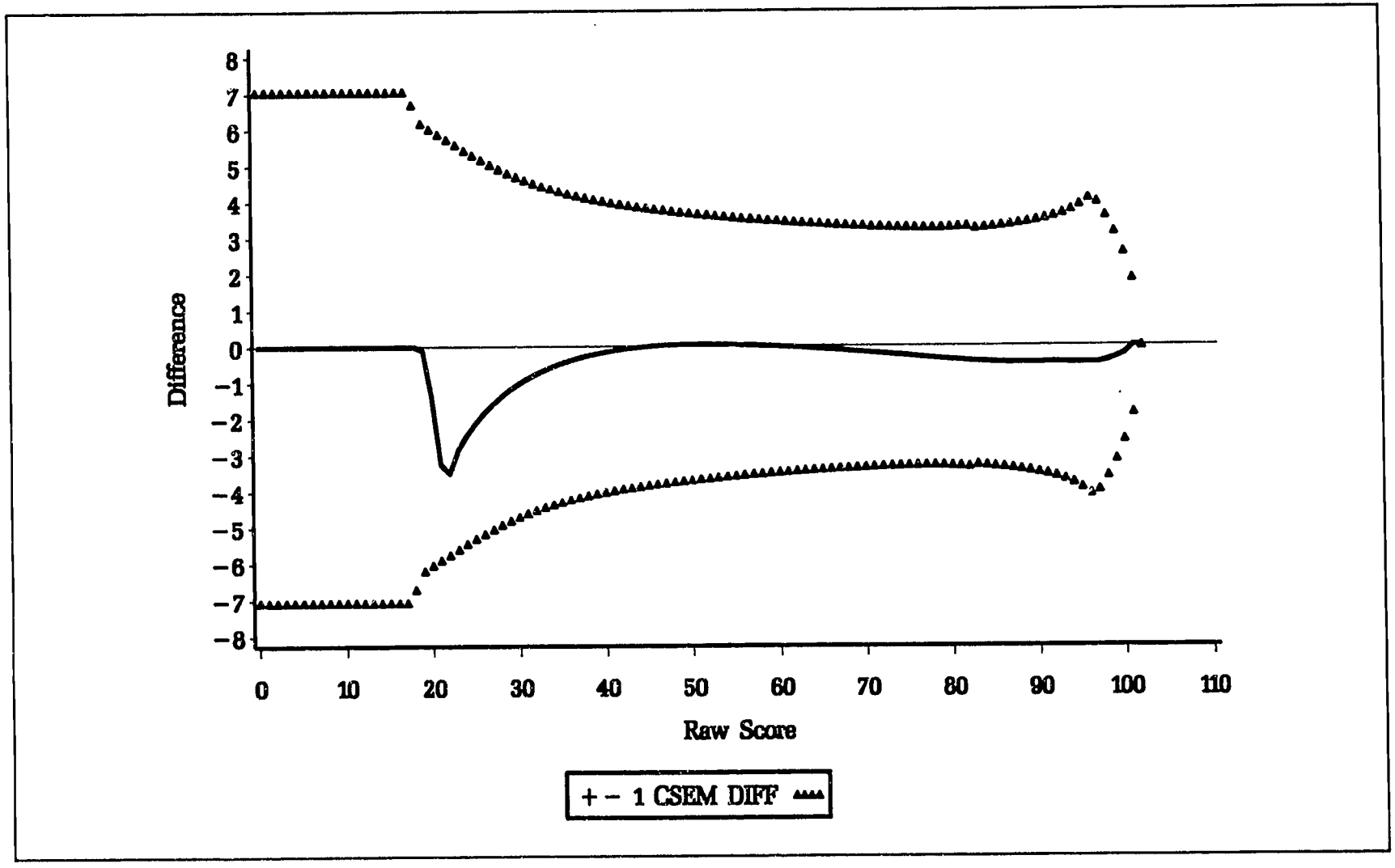

FIGURE 10. Matched Caucasian Sample – African-American Equating Residual Plot

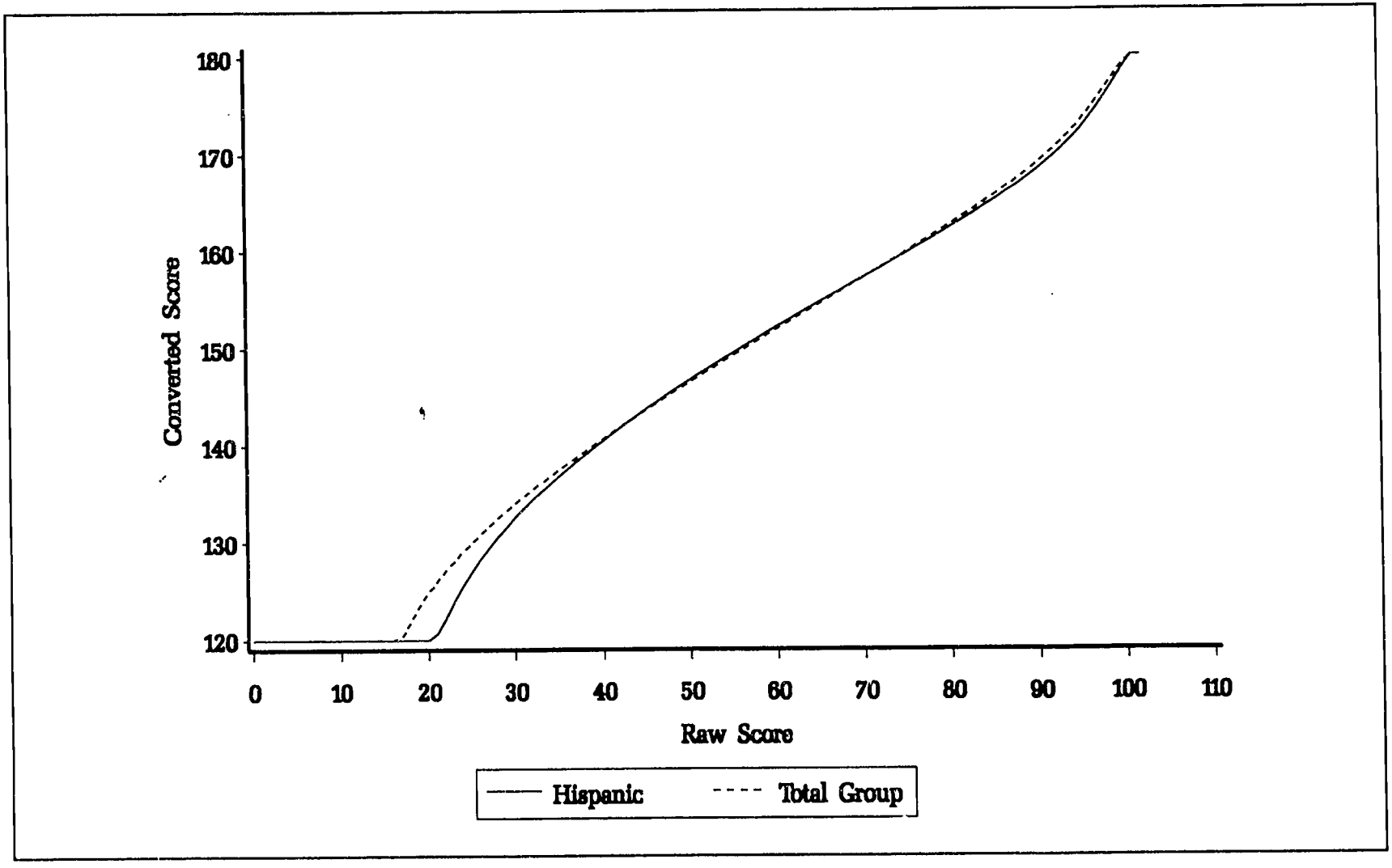

FIGURE 11. LSAT Hispanic and Total Group Raw to Scaled Score Conversion Lines

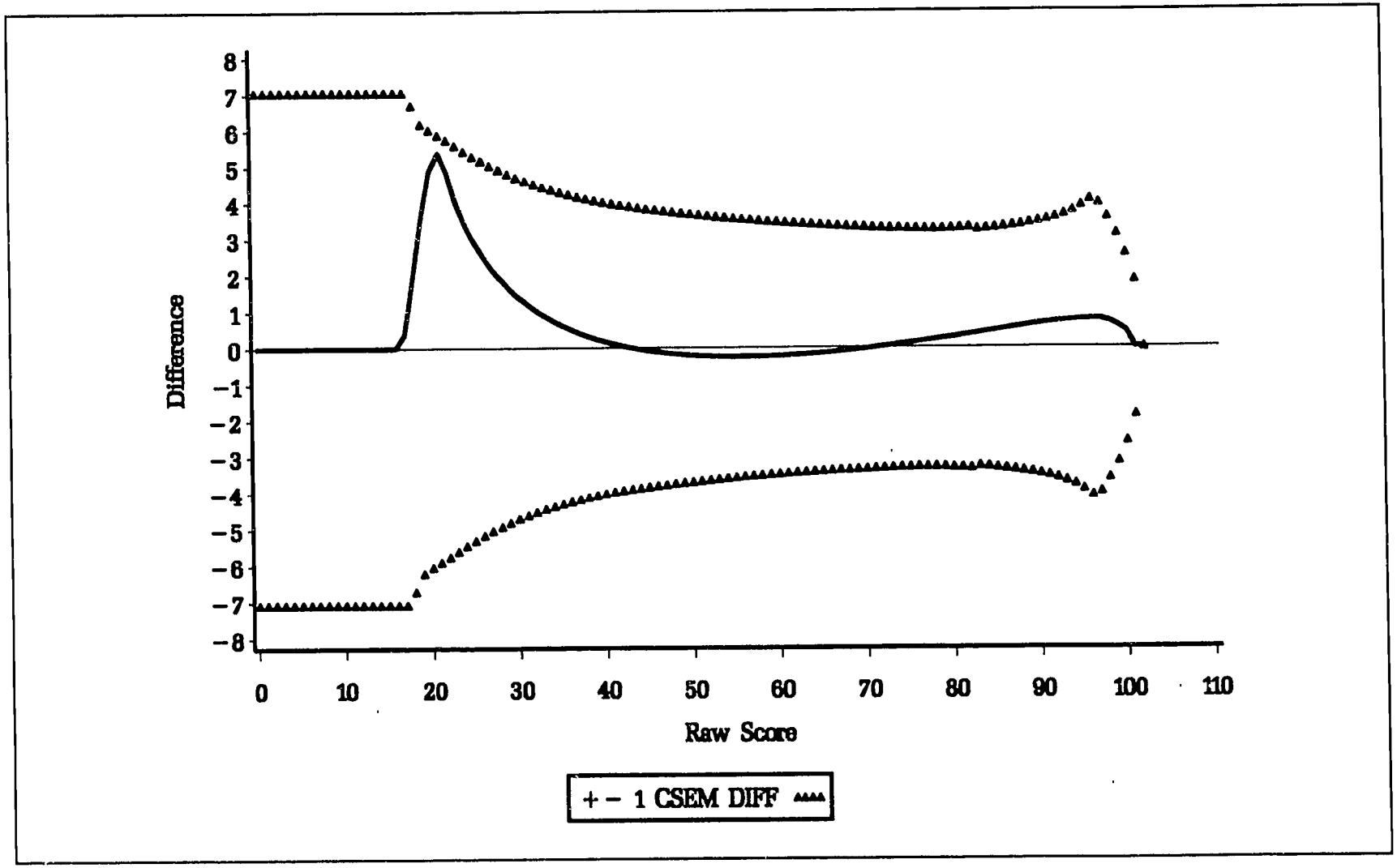

FIGURE 12. Total Group – Hispanic Equating Residual Plot

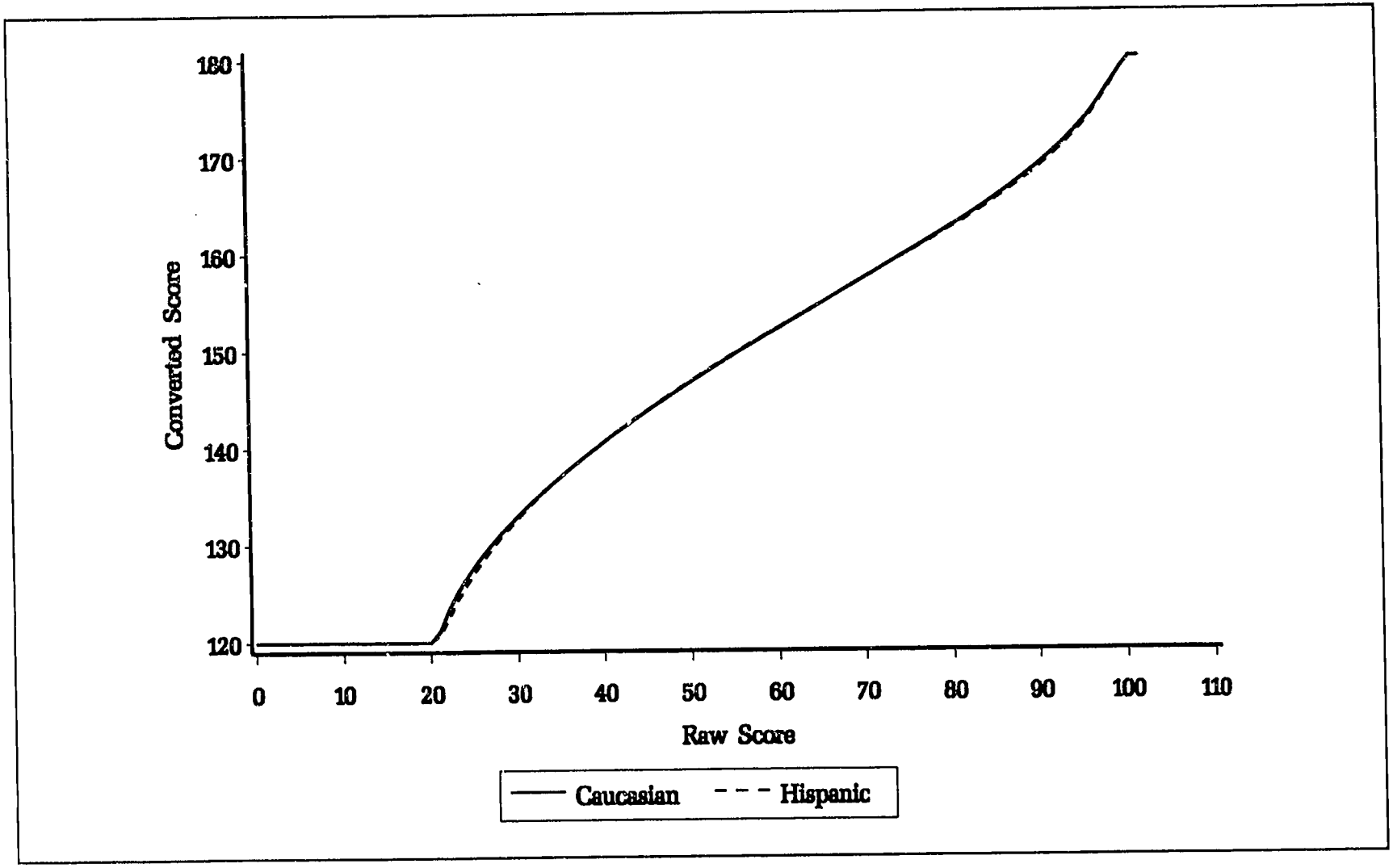

FIGURE 13. Caucasian and Hispanic LSAT Raw to Scaled Score Conversion Lines

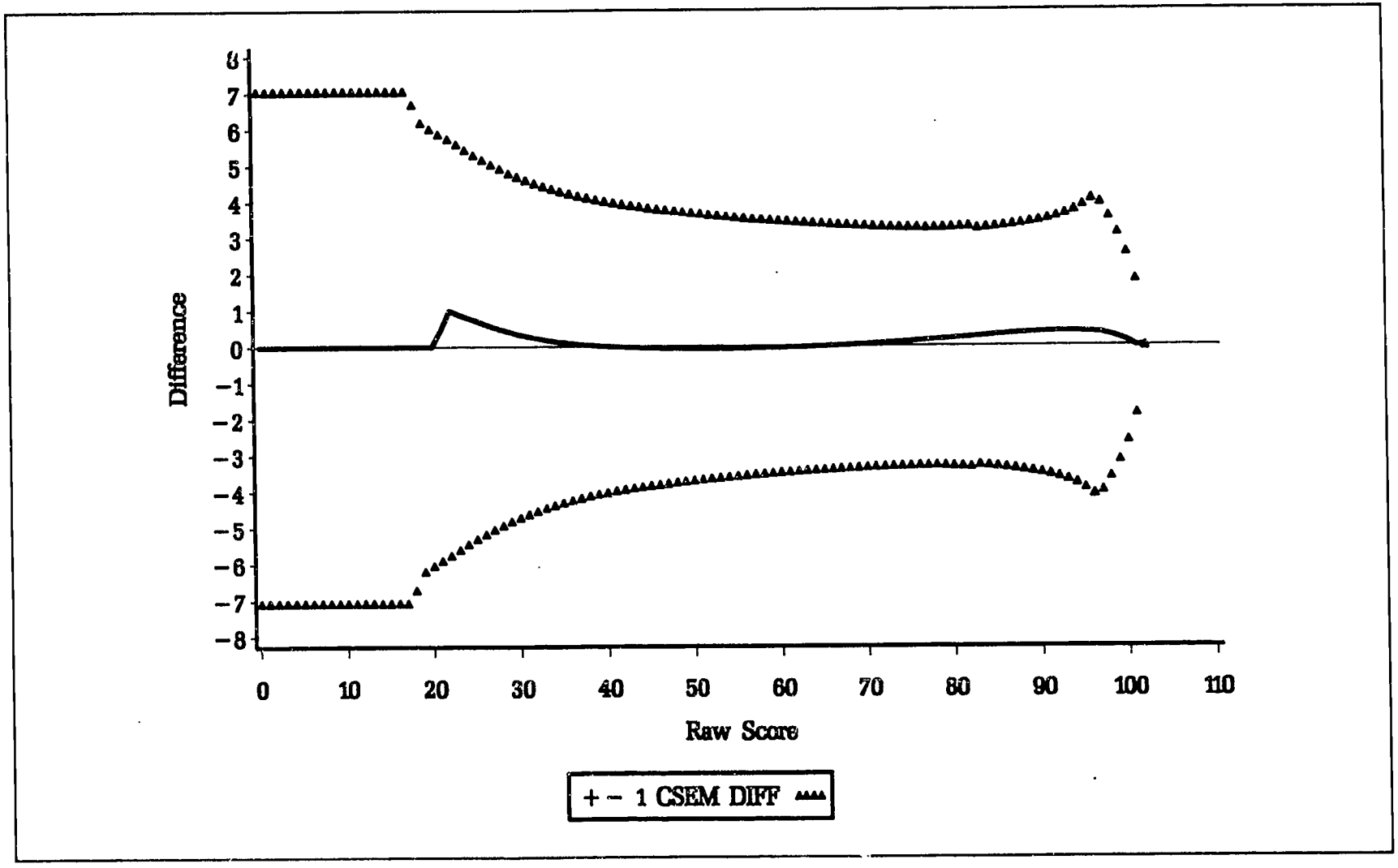

FIGURE 14. Caucasian – Hispanic Equating Residual Plot

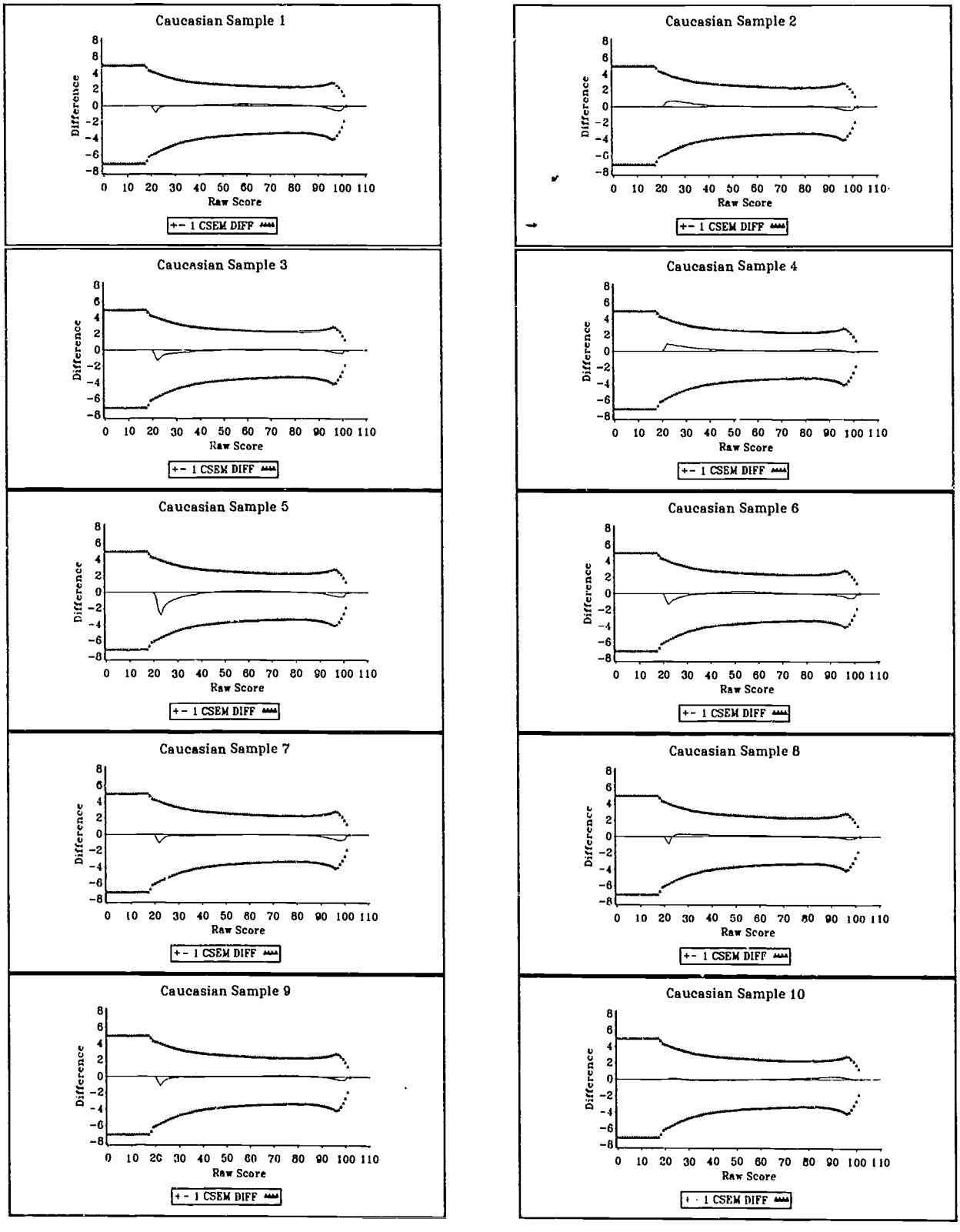

FIGURE 15. Random Sample Caucasian-Hispanic Equating Residual Plots